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COMPARISON OF ANALYTICAL AND NUMERICAL APPROACHES FOR SIMULATING GROUNDWATER FLOW TO MULTI SCREEN WELLS

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Abstract: Groundwater is frequently abstracted from wells with several screens or partially penetrating the aquifer. Such wells play significant role in groundwater production, drainage or remediation. The determination of the hydraulic behavior of wells, which are open to more than one depth zone or have short screen, is rather difficult. In this comparative study, different analytical and numerical methods are reviewed and compared to give a clear idea about the accuracy and reliability of the well-known Multi-Node Well (MNW) package for the widely used USGS code MODFLOW. The obtained results proved that the MNW package can provide acceptable and accurate simulations even in certain particular hydraulic situations. On the other hand the case-study examples also confirmed that the MNW Package cannot be recommended for short-term transient simulations because of approximation error.

Key words: Partially penetrating well, multi screen wells, Multi-Node Well package, MODFLOW, groundwater flow modeling, analytical and numerical simulations

1. INTRODUCTION

Multi screen wells are open across two or more depth zones of aquifers that may have different hydraulic properties and heads. In multilayer aquifer systems these special wells may interconnect several aquifers and can have a profound effect on groundwater flow, regardless of pumping (Neville & Tonkin 2004). Flow meter survey may quantify the yield of each screen unit and, thus, these data can also be used in calibration of groundwater flow models. The modular finite difference groundwater flow model, MODFLOW, developed by the U.S. Geological Survey (USGS) is a computer program for simulating common features in groundwater systems (Harbaugh et al., 2000). The code was constructed in the early 1980's, and has continuously evolved since then with development of many new packages and related programs for groundwater investigations (McDonald & Harbaugh 2003). Recently, MODFLOW is the most widely used program in the world for simulating groundwater flow (Chiang & Kinzelbach 2001).

MODFLOW uses the input to construct and solve equations of groundwater flow in the aquifer system. The solution provides head or groundwater level at every cell in the model. Hydrogeologists usually use water levels from the model layer to draw contour maps for comparison with similar maps prepared from field data. In addition to water levels, MODFLOW prints a water budget for the entire aquifer system. Simulation of pumpage by wells is a fundamental and widely used feature of groundwater models such as MODFLOW (Szucs et al., 2006). Simulation capability of wells in MODFLOW is limited to withdrawal at specified rates from individual cells. The governing partial differential equation solved numerically in MODFLOW is given in the following form:

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)+W=S_S\frac{\partial h}{\partial t} \quad ,(1)$$

where K_{xx}, $K_{yy,}$ and K_{zz} are the values of the hydraulic conductivity along the x, y, and z coordinate axes (L/T), h is the hydraulic head (L), W is the volumetric flux per unit volume representing the sources and sinks of groundwater, for which the negative values denote extractions while the positive values denote injections (T^{-1}), S_S is the specific storage of the investigated aquifer (L^{-1}), and t is time

(T). The pure MODFLOW code is free software, written in the FORTRAN language, and can be compiled and run on the DOS, Windows, or UNIX operating systems.

Real pumpage from aquifers is commonly more complex. Heads in aquifers that surround a well are likely to vary along the length of a screen that penetrate the aquifer or has a long horizontal extent. Because of this complex flow behavior, a computer program called the drawdown-limited, Multi-Node Well (MNW) Package was developed for MODFLOW (Halford & Hanson 2002). The MNW package allows MODFLOW users to simulate wells with short or multiple screens that extend beyond a single model node. Multi-node wells can simulate wells that are completed in multiple aquifers or in a single heterogeneous aquifer, hydraulic effect of partially penetrating (see Fig. 1.), and horizontal wells. The multi-node aspect of the MNW package can enhance model calibration and groundwater capabilities of MODFLOW.

The revised Multi-Node Well (MNW2) Package was launched in 2009 (Konikow et al., 2009). This improved MNW2 module was also used in the comparative case-study examinations.

An alternate numerical multiscreen well flow simulator FLOW was developed by Székely (Székely et al. 1996, 2000 and Székely 2012). The well flow module of FD (finite difference) groundwater flow simulator estimates the well bore drawdown and screen fluxes with the effects of laminar and turbulent skin losses. The point centered FD scheme solves Eq. (1) and generates the cell drawdown due to distributed flux W, whereas the additional local drawdown in well bore is calculated for the actual (confined or leaky) flow conditions around the screen(s). A set of (optionally nonlinear) algebraic equations is solved to get the fluxes of screens yielding uniform drawdown in the well bore. This calculation is performed at all FD iterations until stable flowrate distribution is reached within the time step or over steady state simulation.

Performance of numerical FD simulators MODFLOW and FLOW is evaluated in a detailed manner via comparative testing against 3D analytical simulations under steady-state (section 2) and unsteady (section 3) flow conditions. The purpose of this paper is to demonstrate the applicability, the reliability and the accuracy of the MNW2 and FLOW packages through a series of model simulations under various conditions, which are important for practical investigations.

2. COMPARATIVE SIMULATIONS OF STEADY-STATE DRAWDOWN AND WELLBORE FLOW

2.1. Model description

A composite model has been developed for a vertical well operating in a confined homogeneous isotropic aquifer with user defined boundary conditions along four rectangular straight line boundaries. The well fully or partially penetrates the aquifer and even may exhibit discontinuous screening. The software WT (current version of TEST by Székely 1995) is used for 3D drawdown and flow rate simulations. Line source solution by Hantush (1961) is applied as transition function to define the vertically averaged drawdown along screen (sections).

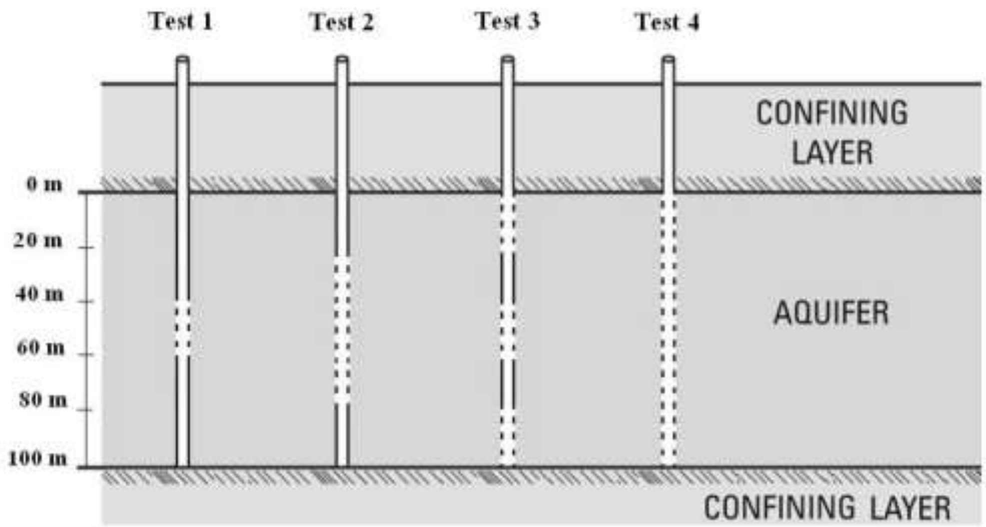

Figure 1. Fully (Test 4), partially (Test 1, Test 2) penetrating and multi screen (Test 3) wells in a confined aquifer (after Konikow et al., 2009).

Uniform well-bore drawdown and depth dependent flow profile are assumed in the well bore. Recently this condition is referred to as Uniform Well-face Drawdown or UWD well flow option (Hemker 1999). This model enables one to simulate both the well bore drawdown and the flow rate variation along screens. The effect of four boundaries is calculated by the method of images (Székely 1990 equations 8-13). The steady-state flow data due to the pair of parallel fix head boundaries are modeled as large time expansion of the unsteady WT simulation.

Four different tests were carried out to demonstrate how the MNW2 and FLOW packages perform concerning the simulation results. Layouts of well screening are shown in figure 1, results of calculations are summarized in table 1. The confined simulation model comprises 5 homogeneous horizontal layers with equal parameters as follows:

Thickness: 20 m.

Initial hydraulic head: 0 m.

The hydraulic conductivity: 0.0001 m/s.

Horizontal and vertical anisotropy: 1.

Specific storage: 0.00001 1/m.

Parameters of the model area:

Left bottom corner coordinate is: (0 m, 0 m).

Right uppermost corner coordinate is: (510 m, 510 m).

Grid system used by MODFLOW: 51 rows, 51 columns. The basic grid size: dl = 10 m.

Well data:

A pumping well is located in the middle of the modeled area. The coordinate is: (255 m, 255 m).

The radius of the pumping well is: 0.2 m.

The discharge rate of the pumping well is: -0.1 m^3/s.

The FLOW package uses 50 blocks in both x and y directions and the square blocks have dl = 10.2 m long sides. Thus, the well can be positioned at the common corners of four neighboring blocks as required by the point centered FD schemes or finite element simulators.

To use the MNW2 package with MODFLOW, the hydrodynamic model of the supposed area was compiled using the Groundwater Modeling System (GMS 8.3) modeling package. The applied grid system with the boundary conditions and the well in the middle can be seen in Figure 2 whereas the vertical layout is presented in figure 3. Specified head boundary conditions were set on the west and east side of the model area. For the sake of proper representation of specified head boundaries exhibiting diminishing width, two additional boundary columns at ΔX = 0.1 m were added. No flow boundary conditions were set in the north and south side of the model area.

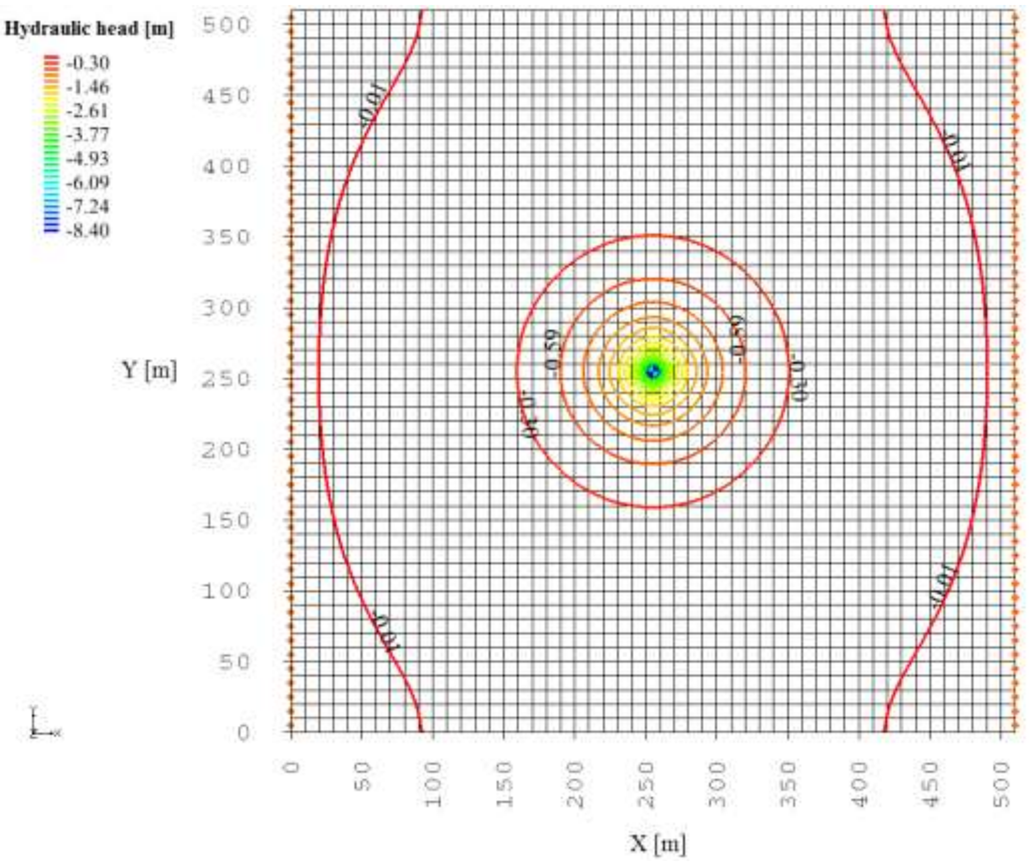

Figure 2.The applied grid system with the boundary conditions, the production well in the middle of the model area and drawdown contour lines of Test 4.

From technical point of view as well as for groundwater management purposes it is important to determine the local drawdown of the investigated production well (Szekely 1975). In case of a finite difference approach like MODFLOW, the local drawdown s_{well} in any particular well is obtained as the sum of the grid drawdown (s_{grid}) and the corrected increment drawdown (ds_{well}). The corrected increment drawdown can be calculated based on the following equation (Peaceman 1983):

$$ds_{well} = \frac{Q}{2\pi T} \ln 0.2 \frac{\Delta X}{r_w} \quad (2)$$ where,

Q – the discharge rate of the pumping well [m^3/s],
T – the average transmissivity of the cell element, where the well is located [m^2/s],
ΔX– the side length of the square shapedcell element (grid), where the well is located [m],
r_w – the actual radius of the pumping well.

Equation (2) considers purely radial flow in the block hosting the screen. It is a reasonable assumption for screens with no or negligible vertical flow from/to the block. Fully penetrating wells in formations with impermeable top and bottom (Test 4) are good example of this flow scheme. Some wells or formations may exhibit different features including partial penetration (Test 2), discontinuous screening (Test 3) or leaky top. These conditions may generate sufficient vertical flow to selected screens and can bias some simulation results obtained by MODFLOW (see Table 1).

The skin effect can be expressed as the change of hydraulic conductivity (or permeability) around productions wells. The MNW2 package enables the modeller to incorporate the skin effect into the simulations. The skin effect can be pictured as head loss occurring across a cylinder of radius, r_{Skin}, around the investigated well with a finite radius, r_w. The skin zone has a transmissivity T_{Skin}, that differs from the formation transmissivity T. The dimensionless skin coefficient can then be described in terms of a transmissivity contrast (T/T_{Skin}) over the finite difference between r_w and r_{Skin} or by:

$$\text{Skin} = \left(\frac{T}{T_{Skin}} - 1\right)\ln\left(\frac{r_{Skin}}{r_w}\right). \quad (3)$$

In most cases the Skin is positive. The skin coefficient is equal to zero or negative if T_{Skin} is equal to or greater than T. The additional (+ or -) drawdown ds_{Skin} caused by the skin effect can be calculated as:

$$ds_{Skin} = \frac{Q}{2\pi T} \cdot \text{Skin}. \quad (4)$$

Tests 1, 2, 3 and 4 (see Fig. 1) were performed at the same rate of 0.1 m^3/s and demonsrate effect of different screening shemes in the 5-layer-model. In case of Test 1 only one layer was screened. This scheme may involve five different options, as the screen can be installed separately at the top (in layer 1), at the bottom (in layer 5), in the middle (in layer 3), and in layers 2 or 4. In case of Test 2 layers 2, 3 and 4 are screened, whereas layers 1, 3 and 5 are tapped in Test 3. Finally, in case of Test 4 all the five layers are screened simultaneously.

Konikow et al., (2009) conducted a detailed study on partially penetrated wells. They concluded that beyond a certain elapsed time MODFLOW simulation of drawdown evolution can be performed at reasonable accuracy. Results are presented in graphical form. In the present comparative study both the steady-state drawdown s [m] in the well bore and the flow rate contribution Q% of screens were calculated, data are given in the table 1.

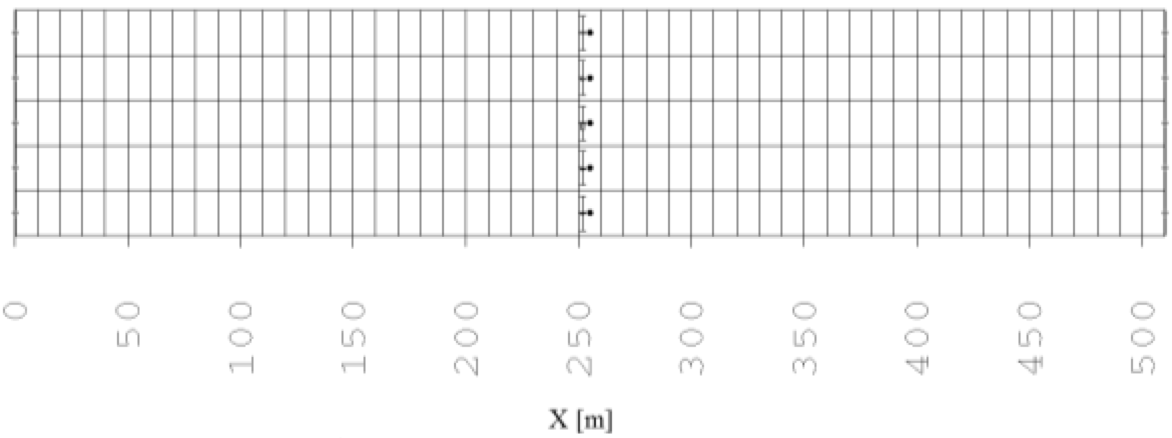

Figure 3. The cross-section of the 5-layer-model for Test 1, 2, 3 and 4.

2.2. Evaluation of the results

Results of comparative simulations are summarized in table 1. Column 4 displays 3D_A data obtained by WT simulation considering 5 model layers and l_{scr} = 20 m long screens represented by line sinks. Column 5 exhibits results of an enhanced 3D_B modeling where each screen is split into 80 sections of l_{scr} = 0.25 m. This segmentation provides a very detailed flux distribution along screens therefore these data are considered as the "true" solution to the test problem analyzed and used as reference values. The last two columns show results of numerical MODFLOW and FLOW simulations with close s data and higher discrepancy in calculated fluxes.

For the other tests numerical well bore drawdown data show close overestimation. This, however, can be reduced through vertical refinement of the 100 m thick flow domain. Thus, by applying 50 model layers and 10 sub-screens (l_{scr} = 2 m) in the upper 20 m thick section the first s value in the last column reduces to 40.226 m. The latter is a close approximation to the 3D_B simulation. Closer inspection of Q% data reveals that the 3D_B fluxes (reference data) are positioned between the two numerical solutions.

The drawdown in and the flux into/from the well bore is sufficiently controlled by dimensionless parameter Skin. Thus, introduction of appropriate skin parameters enables one to eliminate the discrepancy between the 3D_B and numerical (MODFLOW, FLOW) well bore simulations. Table 2 exhibits results of Skin estimation.

In case of Test 1 the Skin data of separate aquifers have been obtained by means of forward calculation using Equation 4 with $ds_{Skin} = s_{analytical} - s_{numerical}$. In case of Tests 2 and 3 the Skin data have been defined through manual (MODFLOW) or automatic (FLOW) calibration. Fitting of Q% data to theoretical values or field measurements (flow metering) is of high importance for appropriate solute transport modeling involving multi screened wells (Konikow et al., 2009).

3. UNSTEADY DRAWDOWN SIMULATION: THE FAIRBORN PUMPING TEST

The MODFLOW and FLOW software are frequently used for unsteady well flow problems. The MNW or the MW2 module of MODFLOW is not recommended for short term transient effects (Halford & Hanson 2002, Konikow et al., 2009), by contrast, FLOW has no similar constraints on simulation time. In this section the temporal performance of the software is evaluated, the Fairborn pumping test (Lohman 1979) is selected for comparative numerical analysis.

Table 1. Summary of comparative evaluation of well flow simulators.

Tests	**Screened layers**	**Data**	**3D_A l_{scr}=20 m**	**3D_B l_{scr}=0.25 m**	**MODFLOW dl=10 m**	**FLOW dl=10.2 m**
1	1 or 5	s	40.648	39.986	42.087	42.030
	2 or 4		36.608	35.822	39.274	39.212
	3		36.275	35.486	38.950	38.887
2	2-3-4	s	16.409	16.253	16.730	16.730
		Q% 2 & 4	34.578	34.577	34.872	34.001
		Q% 3	30.843	30.846	30.245	31.999
3	1-3-5	s	15.724	15.507	16.305	16.304
		Q% 1 & 5	32.284	32.223	32.071	32.727
		Q% 3	35.432	35.554	35.855	34.547
4	1-2-3-4-5	s	12.053		12.052	12.053

Table 2. Skin parameters required to fit numerical s and Q% data to the analytical model 3D_B.

Tests	**Screened layers**	**Skin**	
		MODFLOW	**FLOW**
1	1 or 5	-0.2641	-0.3534
	2 or 4	-0.4337	-0.5466
	3	-0.4352	-0.5479
2	2 & 4	-0.1065	-0.2374
	3	0.10624	0.0412
3	1 & 5	0.101	-0.3358
	3	-0.0966	-0.5252

3.1. The field test and the aquifer parameters

The aquifer test (Lohman 1979) was conducted near Dayton (Ohio) and involves the pumping (No. 140) and the observation (No. 139) wells drilled at distance of 22.25 m in a water table aquifer. The 25.91m thick presumably homogeneous and anisotropic aquifer is composed of glacial sand and gravel. The wells fully penetrate the aquifer, the radius r_w are 0.2286 m and 0.0762 m for the pumping and observation wells, respectively. The well 140 was pumped at a high steady rate of 0.068206 m^3/s. Drawdown data are available from the well 139 only (Figure 4 displays the measured data).

The software WT was used to estimate the aquifer parameters. The analytical method assumes 3D flow in the formation and uses the concept of segmented or layered aquifer to assess vertical variation of both heads and fluxes (see section 2). The following 13 segments have been introduced: a) 1 m thick water table top layer, b) 0.71 and 0.70 m thick layers underlying the top layer and located above the simulated dynamic level in the pumping well and c) 10 segments of 2.35 m thickness to simulate the tapped section. The model assumes that the upper three segments are not discharged by the pumping well.

The UWD flow model is applied to the pumping well (140). The observation well (139) is considered as an idle well of uniform well bore drawdown discharged and recharged at different depth zones depending on the vertical variation of external drawdown caused by the pumping well. The external drawdown values are calculated without the hydraulic effect of the high conductivity well bore. Secondly, this time and depth variant drawdown function is used as the external stress, causing induced inflow and outflow in the unpumped observation well. This well exhibits transient but vertically uniform well bore drawdown. The outlined induced flow controlled drawdown (IFD) simulation technique is described, tested and applied to this field experiment by Székely (2012). The computer calibration yielded the following parameters: $K_{xx} = K_{yy} = 1.3657\times10^{-3}$ m/s, $K_{zz} = 8.8954\times10^{-5}$ m/s, $S_s = 0.6851\times10^{-4}$ 1/m, $S_y = 0.13285$ (S_y denotes the specific yield). Zero skin loss is assumed in both wells.

3.2. MODFLOW and FLOW simulations

The above vertical segmentation and parameters were used for numerical simulation. The MODFLOW and FLOW models utilized a square shaped model area. The lateral extension of the flow domain was selected sufficiently large to have negligible drawdown at no-flow boundaries. This is necessary to avoid false impact of artificial boundaries required by numerical models. The realization of the MODFLOW grid can be seen in figure 5. As it was mentioned earlier, finer grid size was applied around the pumping well.

The well-bore storage coefficient πr_w^2 affects the early time drawdown evolution and is included into S_s parameter of blocks incorporating the screens.

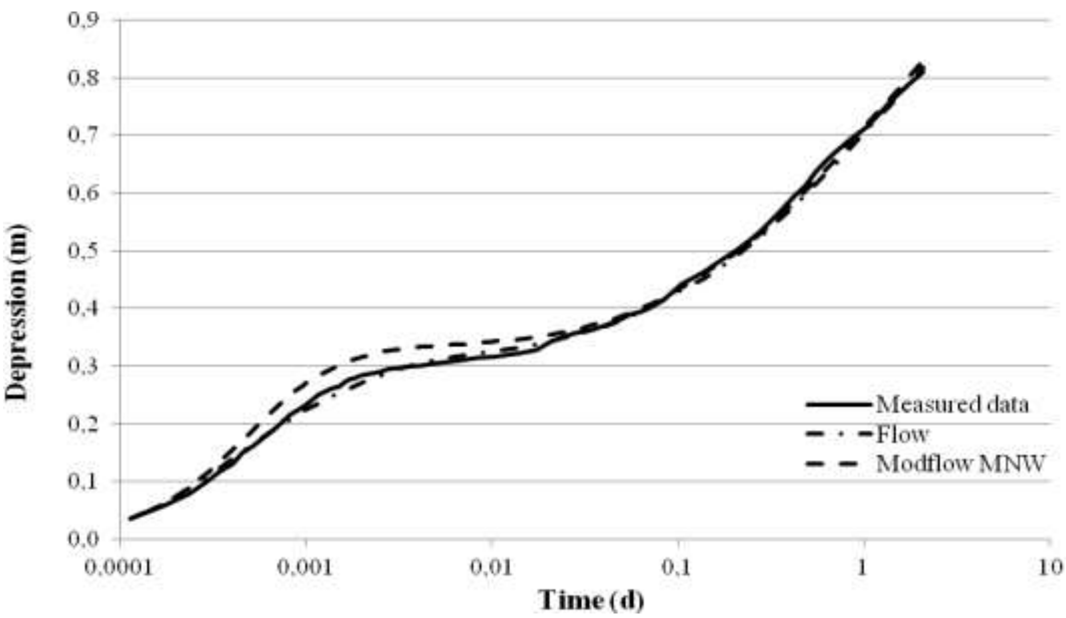

Figure 4. The measured and simulated depression data for the Fairborn pumping test.

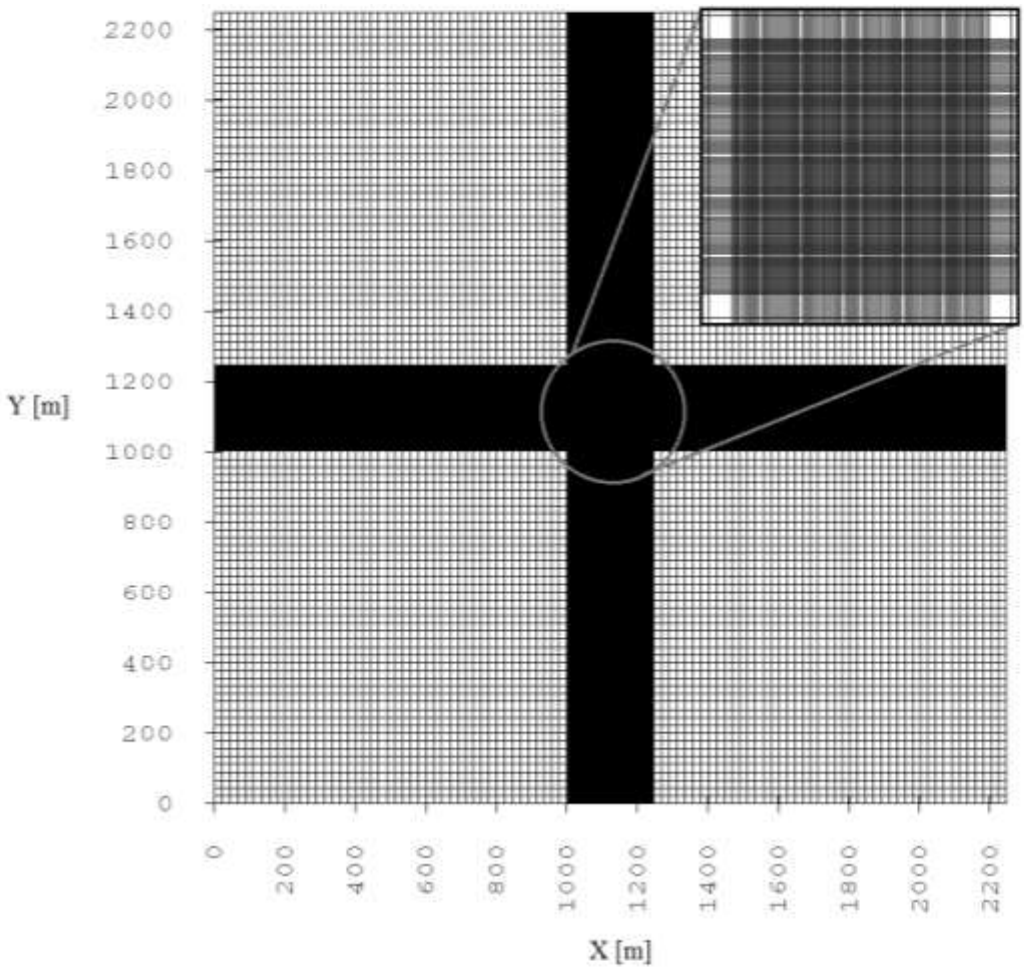

Figure 5. The realization of the MODFLOW grid Fairborn pumping test.

The calculated drawdown-time curve is presented in Figure 4. At early time the simulated MODFLOW-MNW data deviate from the measured drawdown as stated by Halford & Hanson (2002) and shown by Konikow et al., (2009). At elapsed time $t > 0.07$ d (that is under quasi-steady state flow conditions) the simulated and measured data show small difference.

The FLOW simulation in Figure 4 exhibits reasonable fit over the whole simulation time $0 \leq t \leq 2.0833$ d. Thus this software can be used for well test data analysis involving short time measurements (Székely 2012).

The upper part of the well bore discharges whereas the lower part recharges the aquifer in the observation well. The analytical WT simulation resulted in low $\pm 3.544\times10^{-4}$ m^3/s flux entering and leaving the well bore at the end of pumping. The effect of well-bore storage is negligible at this time. The numerical FLOW software yielded close induced flow of $\pm 3.728\times10^{-4}$ m^3/s. The MODFLOW-MNW simulation concluded with induced flux of $\pm 5.688\times10^{-4}$ m^3/s. The higher value is caused by the more approximate way of calculating the well-bore drawdown and flux of screens.

4. CONCLUSIONS

The following conclusions can be made on the basis of the present study:

1. Two analytical (3D_A, 3D_B) and two numerical (MODFLOW, FLOW) methods were applied and compared to test their multiaquifer well flow simulation abilities.
2. The obtained results proved that under steady-state flow conditions the numerical MODFLOW MNW and FLOW packages can provide acceptable and accurate simulations even in complex hydraulic situations in multilayer aquifers.
3. In coherence with conclusion by Konikow et al., (2009) results of the case-study example confirm that the MNW package has a certain discrepancy when simulating short term transient effects. On the other hand, the MNW package provides accurate and reliable simulation results if the elapsed time is longer than 1.5 hour.

4. In case of multiscreen well flow simulation several modeling techniques are recommended to establish and minimize the approximation error of different origin and range. This may help in finding the optimum solution to evaluate flow metering data, contaminant transport and to involve numerical simulation techniques into model calibration.

5. ACKNOWLEDGMENTS

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